

# Generation of concatenated Greenberger-Horne-Zeilinger-type entangled coherent state based on linear optics

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The concatenated Greenberger-Horne-Zeilinger (C-GHZ) state is a new type of multipartite entangled state, which has potential application in future quantum information. In this paper, we propose a protocol of constructing arbitrary C-GHZ entangled state approximatively. Different from the previous protocols, each logic is encoded in the coherent state. This protocol is based on the linear optics, which is feasible in experimental technology. This protocol may be useful in quantum information based on the C-GHZ state.

## I. INTRODUCTION

Quantum information, based on quantum mechanics theory, provides a great opportunity to develop computer and communication technology. Based on bipartite entanglement, many important quantum information protocols were developed, such as quantum computation [1], quantum teleportation [2], quantum key distribution [3], quantum secure direct communication [4, 5], dense coding [6] and other significant quantum protocols [7–11]. Besides the bipartite entanglement, multipartite entanglement also plays an important role in quantum information, such as controlled teleportation [12], quantum state sharing [13, 14] and quantum secret sharing [15]. Among different types of multipartite entanglement, the Greenberger-Horne-Zeilinger (GHZ) state has been widely used [16]. However, due to the increase of the number of particle in noisy environment, the GHZ state will become fragile. Recently, Fröw and Dür discussed a new type of multipartite entanglement, called concatenated Greenberger-Horne-Zeilinger (C-GHZ) state [17]. The C-GHZ state essentially is the logic-qubit entanglement and it can be written as

$$|\varphi\rangle_{N,m} = \frac{1}{\sqrt{2}}(|GHZ_m^+\rangle^{\otimes N} + |GHZ_m^-\rangle^{\otimes N}), \quad (1)$$

with  $|GHZ_m^\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle^{\otimes m} \pm |1\rangle^{\otimes m})$ , where  $N$  is the number of logical qubits. Each logic qubit is encoded by  $m$  physical qubits. Fröw and Dür showed that the C-GHZ state not only has the similar feature of GHZ state, but also has a better robustness in noisy environment [18]. Subsequently, there are some interesting work based on the C-GHZ state, such as the generation of C-GHZ state with cross-Kerr nonlinearity [19], the Bell-state analysis for C-GHZ state [20–23], entanglement purification [24] and concentration [25, 26]. Based on the linear optics, Lu *et al.* designed the first experiment realization for C-GHZ state, and showed that such state may has its potential application in future quantum information area [27].

On the other hand, in quantum information, there are two different approaches to encode the qubit. The first is the discrete variables (DV), such as the polarization states of photons [28], the spatial modes of the photons, and so on. The second is the continuous variables (CV), such as the squeezed-state entanglement, Gaussian state and coherent state [29–50]. For example, it is shown that the squeezed state can be used in quantum teleportation, QKD and some other important quantum communication protocols [29–33]. In 2016, Faria described an optical approach for Gaussian state to verify bipartite entanglement. The most advantage is that this protocol does not destroy both systems and their entanglement is proposed [35]. The entangled coherent state (ECS) is another important type of CV entanglement, which will be detailed here. In 2001, Wang *et al.* described the quantum teleportation based on the ECS and showed that the ECS can also be used in quantum communication [38]. In 2002, Jeong *et al.* described

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an important protocol for quantum computation with ECS [39]. Recently, the entanglement concentration for both Bell-type and W-type ECS were proposed [44, 45].

Though several quantum protocols for C-GHZ state were proposed, they all focus on the state encoded in DV, none protocol discuss the C-GHZ state in the framework of CV. In this paper, we will propose the first approach to prepare the C-GHZ state encoded in coherent state approximatively, which is named C-GHZ-type ECS. The qubit of C-GHZ-type ECS is encoded in coherent state, i. e.,  $|0\rangle \equiv |\alpha\rangle$  and  $|1\rangle \equiv |-\alpha\rangle$ , respectively. Each logic qubit is an ECS. This protocol is based on linear optics and may be feasible in future experiment. This paper is organized as follows. In section 2, we briefly describe the protocol of constructing C-GHZ-type ECS with  $N = m = 2$  and  $N = 2, m = 3$ , respectively. In section 3, we extend this protocol to the case C-GHZ-type ECS with arbitrary  $N$  and  $m$ . In Section 4, we provide discussion and conclusion.

## II. GENERATION OF C-GHZ-TYPE ECS WITH $N = m = 2$ AND $N = 2, m = 3$

In this section, we will propose a simple protocol to generate the C-GHZ-type ECS approximatively. We first describe the approach to generate the C-GHZ-type ECS with  $N = m = 2$ . The C-GHZ-type ECS is

$$|\psi\rangle_{2,2} = \frac{1}{\sqrt{2}}(|GHZ_2^+\rangle^{\otimes 2} + |GHZ_2^-\rangle^{\otimes 2}) \quad (2)$$

Here, state  $|GHZ_2^\pm\rangle = [2(1 \pm e^{-4|\alpha|^2})]^{-\frac{1}{2}}(|\alpha\rangle_1|\alpha\rangle_2 \pm |-\alpha\rangle_1|-\alpha\rangle_2)$ . States  $|GHZ_2^\pm\rangle$  are the Bell-type ECS.

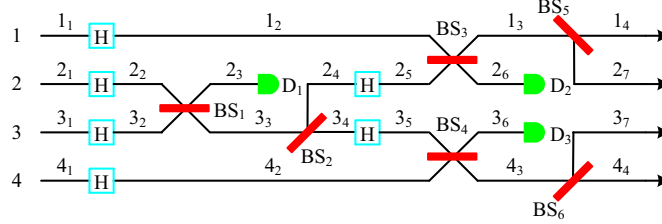


FIG. 1: Schematic diagram of generating the C-GHZ-type ECS with  $N = m = 2$ .  $H$  represents the Hadamard operation which makes  $|\alpha\rangle \rightarrow \frac{N_0}{\sqrt{2}}(|\alpha\rangle + |-\alpha\rangle)$  and  $|-\alpha\rangle \rightarrow \frac{N'_0}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle)$ . The BS is 50:50 beam splitter.

As shown in Fig.1, firstly, we prepare four coherent states  $|\alpha\rangle$  in the spatial modes  $1_1, 2_1, 3_1$  and  $4_1$ , respectively. The four coherent states are  $|\alpha\rangle_{1_1}, |\alpha\rangle_{2_1}, |\alpha\rangle_{3_1}$  and  $|\alpha\rangle_{4_1}$ , respectively. Subsequently, we perform a Hadamard operation on each single coherent state [50]. The Hadamard operation can make  $|\alpha\rangle \rightarrow \frac{N_0}{\sqrt{2}}(|\alpha\rangle + |-\alpha\rangle)$  and  $|-\alpha\rangle \rightarrow \frac{N'_0}{\sqrt{2}}(|\alpha\rangle - |-\alpha\rangle)$ , respectively. Here, the normalized coefficient  $N_0 = (1 + e^{-2|\alpha|^2})^{-\frac{1}{2}}$  and  $N'_0 = (1 - e^{-2|\alpha|^2})^{-\frac{1}{2}}$ . The state in spatial mode  $2_2$  combined with the state in spatial mode  $3_2$  can be written as

$$\begin{aligned} |\psi\rangle_{1_1} &= \frac{1}{\sqrt{2}}N_0(|\alpha\rangle_{2_2} + |-\alpha\rangle_{2_2}) \otimes \frac{1}{\sqrt{2}}N_0(|\alpha\rangle_{3_2} + |-\alpha\rangle_{3_2}) \\ &= \frac{1}{2}N_0^2(|\alpha\rangle_{2_2}|\alpha\rangle_{3_2} + |-\alpha\rangle_{2_2}|\alpha\rangle_{3_2} + |\alpha\rangle_{2_2}|-\alpha\rangle_{3_2} + |-\alpha\rangle_{2_2}|-\alpha\rangle_{3_2}). \end{aligned} \quad (3)$$

Then, we let the photons in the spatial modes  $2_2$  and  $3_2$  pass through the 50:50 beam splitter (BS). The BS can transform two different coherent states  $|\alpha\rangle$  and  $|\beta\rangle$  as

$$|\alpha\rangle|\beta\rangle \rightarrow \left|\frac{\alpha+\beta}{\sqrt{2}}\right\rangle\left|\frac{\alpha-\beta}{\sqrt{2}}\right\rangle. \quad (4)$$

After passing through the BS<sub>1</sub>, state  $|\psi\rangle_{1_1}$  becomes

$$|\psi\rangle_{1_2} = \frac{1}{2}N_0^2(|\sqrt{2}\alpha\rangle_{2_3}|0\rangle_{3_3} + |-\sqrt{2}\alpha\rangle_{2_3}|0\rangle_{3_3} + |0\rangle_{2_3}|\sqrt{2}\alpha\rangle_{3_3} + |0\rangle_{2_3}|-\sqrt{2}\alpha\rangle_{3_3}). \quad (5)$$

By choosing the cases that the spatial mode 2<sub>3</sub> has no photon, state  $|\psi\rangle_{1_2}$  becomes

$$|\psi\rangle_{1_2} \rightarrow |\psi\rangle_{1_3} = N_1(|0\rangle_{2_3}|\sqrt{2}\alpha\rangle_{3_3} + |0\rangle_{2_3}|\alpha\rangle_{3_3}). \quad (6)$$

Here  $N_1 = [2(1 + e^{-4|\alpha|^2})]^{-\frac{1}{2}}$ . Then we let the photons in the spatial mode 3<sub>3</sub> pass through BS<sub>2</sub>, and then the state  $|\psi\rangle_{1_2}$  will become

$$|\psi\rangle_{1_4} = N_1(|\alpha\rangle_{2_4}|\alpha\rangle_{3_4} + |-\alpha\rangle_{2_4}|-\alpha\rangle_{3_4}). \quad (7)$$

Then, we perform a Hadamard operation on the spatial modes 2<sub>4</sub> and 3<sub>4</sub>, respectively. The state  $|\psi\rangle_{1_4}$  will evolve to

$$|\psi\rangle_{1_5} = \frac{1}{2}N_1N_0^2[(|\alpha\rangle_{2_5} + |-\alpha\rangle_{2_5})(|\alpha\rangle_{3_5} + |-\alpha\rangle_{3_5})] + \frac{1}{2}N_1N_0'^2[(|\alpha\rangle_{2_5} - |-\alpha\rangle_{2_5})(|\alpha\rangle_{3_5} - |-\alpha\rangle_{3_5})]. \quad (8)$$

The states in spatial modes 1<sub>2</sub> and 4<sub>2</sub> combined with  $|\psi\rangle_{1_5}$  can be rewritten as

$$\begin{aligned} |\psi\rangle_{1_6} &= |\psi\rangle_{1_5} \otimes \frac{1}{\sqrt{2}}N_0(|\alpha\rangle_{1_2} + |-\alpha\rangle_{1_2}) \otimes \frac{1}{\sqrt{2}}N_0(|\alpha\rangle_{4_2} + |-\alpha\rangle_{4_2}) \\ &= \frac{1}{4}N_0^4N_1[(|\alpha\rangle_{1_2} + |-\alpha\rangle_{1_2})(|\alpha\rangle_{2_5} + |-\alpha\rangle_{2_5})(|\alpha\rangle_{3_5} + |-\alpha\rangle_{3_5})(|\alpha\rangle_{4_2} + |-\alpha\rangle_{4_2})] \\ &\quad + \frac{1}{4}N_0^2N_0'^2N_1[(|\alpha\rangle_{1_2} + |-\alpha\rangle_{1_2})(|\alpha\rangle_{2_5} - |-\alpha\rangle_{2_5})(|\alpha\rangle_{3_5} - |-\alpha\rangle_{3_5})(|\alpha\rangle_{4_2} + |-\alpha\rangle_{4_2})]. \end{aligned} \quad (9)$$

We let the photons in the spatial modes 1<sub>2</sub> and 2<sub>5</sub> pass through BS<sub>3</sub>, and the photons in the spatial modes 3<sub>5</sub> and 4<sub>2</sub> pass through BS<sub>4</sub>, respectively. The state  $|\psi\rangle_{1_6}$  can be written as

$$\begin{aligned} |\psi\rangle_{1_7} &= \frac{1}{4}N_0^4N_1[(|\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_6} + |-\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_6} + |0\rangle_{1_3}|\sqrt{2}\alpha\rangle_{2_6} + |0\rangle_{1_3}|-\sqrt{2}\alpha\rangle_{2_6}) \\ &\quad \otimes (|\sqrt{2}\alpha\rangle_{3_6}|0\rangle_{4_3} + |-\sqrt{2}\alpha\rangle_{3_6}|0\rangle_{4_3} + |0\rangle_{3_6}|\sqrt{2}\alpha\rangle_{4_3} + |0\rangle_{3_6}|-\sqrt{2}\alpha\rangle_{4_3})] \\ &\quad + \frac{1}{4}N_0^2N_0'^2N_1[(|\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_6} - |-\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_6} - |0\rangle_{1_3}|\sqrt{2}\alpha\rangle_{2_6} + |0\rangle_{1_3}|-\sqrt{2}\alpha\rangle_{2_6}) \\ &\quad \otimes (|\sqrt{2}\alpha\rangle_{3_6}|0\rangle_{4_3} - |-\sqrt{2}\alpha\rangle_{3_6}|0\rangle_{4_3} + |0\rangle_{3_6}|\sqrt{2}\alpha\rangle_{4_3} - |0\rangle_{3_6}|-\sqrt{2}\alpha\rangle_{4_3})]. \end{aligned} \quad (10)$$

By selecting the case that both spatial modes 2<sub>6</sub> and 3<sub>6</sub> have no photon, state  $|\psi\rangle_{1_7}$  will collapse to

$$\begin{aligned} |\psi\rangle_{1_8} &= \frac{1}{4}N_0^4N_1[(|\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_6} + |-\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_6})(|0\rangle_{3_6}|\sqrt{2}\alpha\rangle_{4_3} + |0\rangle_{3_6}|-\sqrt{2}\alpha\rangle_{4_3})] \\ &\quad + \frac{1}{4}N_0^2N_0'^2N_1[(|\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_6} - |-\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_6})(|0\rangle_{3_6}|\sqrt{2}\alpha\rangle_{4_3} - |0\rangle_{3_6}|-\sqrt{2}\alpha\rangle_{4_3})]. \end{aligned} \quad (11)$$

Finally, we let the photons in spatial mode 1<sub>3</sub> pass through BS<sub>5</sub>, and let the photons in spatial mode 4<sub>3</sub> pass through BS<sub>6</sub>. We can ultimately obtain the C-GHZ-type ECS in the spatial modes 1<sub>4</sub>, 2<sub>7</sub>, 3<sub>7</sub> and 4<sub>4</sub> as

$$\begin{aligned} |\psi\rangle_{1_9} &= \frac{1}{4}N_0^4N_1(|\alpha\rangle_{1_4}|\alpha\rangle_{2_7} + |-\alpha\rangle_{1_4}|-\alpha\rangle_{2_7})(|\alpha\rangle_{3_7}|\alpha\rangle_{4_4} + |-\alpha\rangle_{3_7}|-\alpha\rangle_{4_4}) \\ &\quad + \frac{1}{4}N_0^2N_0'^2N_1(|\alpha\rangle_{1_4}|\alpha\rangle_{2_7} + |-\alpha\rangle_{1_4}|-\alpha\rangle_{2_7})(|\alpha\rangle_{3_7}|\alpha\rangle_{4_4} - |-\alpha\rangle_{3_7}|-\alpha\rangle_{4_4}) \\ &\approx \frac{1}{\sqrt{2}}(|GHZ_2^+\rangle^{\otimes 2} + |GHZ_2^-\rangle^{\otimes 2}). \end{aligned} \quad (12)$$

From Eq. (12), in order to obtain the state in Eq. (2), if  $\alpha$  is large enough, which makes  $e^{-2|\alpha|^2} \rightarrow 0$ .

This approach can be extend to the case of generating the C-GHZ-type ECS with  $N = 2$ , and  $m = 3$  of the form

$$|\psi\rangle_{2,3} = \frac{1}{\sqrt{2}}(|GHZ_3^+\rangle^{\otimes 2} + |GHZ_3^-\rangle^{\otimes 2}). \quad (13)$$

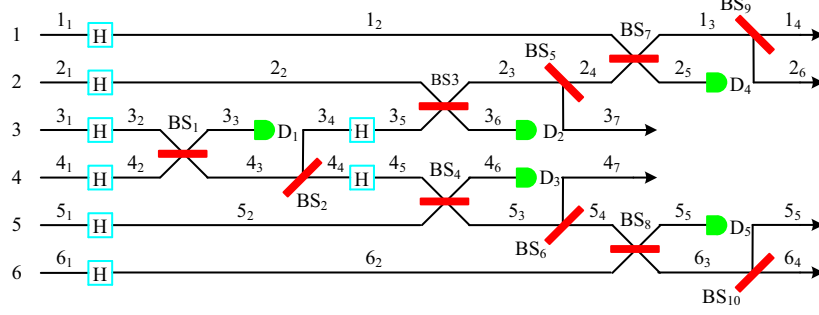


FIG. 2: Schematic diagram of generating the C-GHZ-type ECS with  $m = 3$  and  $N = 2$ .

Here,  $|GHZ_3^\pm\rangle = [2(1 \pm e^{-6|\alpha|^2})]^{-\frac{1}{2}}(|\alpha\rangle_1|\alpha\rangle_2|\alpha\rangle_3 \pm |-\alpha\rangle_1|-\alpha\rangle_2|-\alpha\rangle_3)$ . From Eq. (13), in each logic qubit, it is a GHZ-type ECS. As shown in Fig.2, we first prepare six coherent states  $|\alpha\rangle$  in spatial modes  $1_1, 2_1, 3_1, 4_1, 5_1$  and  $6_1$ . With the same principle shown in Fig. 1, we first generate the state

$$\begin{aligned} |\psi\rangle_{2_1} \rightarrow & \frac{1}{\sqrt{2}} N_1'^2 [ (|\alpha\rangle_{2_4}|\alpha\rangle_{3_7} + |-\alpha\rangle_{2_4}|-\alpha\rangle_{3_7}) (|\alpha\rangle_{4_7}|\alpha\rangle_{5_4} + |-\alpha\rangle_{4_7}|-\alpha\rangle_{5_4}) ] \\ & + \frac{1}{\sqrt{2}} N_1'^2 [ (|\alpha\rangle_{2_4}|\alpha\rangle_{3_7} - |-\alpha\rangle_{2_4}|-\alpha\rangle_{3_7}) (|\alpha\rangle_{4_7}|\alpha\rangle_{5_4} - |-\alpha\rangle_{4_7}|-\alpha\rangle_{5_4}) ]. \end{aligned} \quad (14)$$

in the spatial modes  $2_4, 3_7, 4_7$  and  $5_4$ , which has the same form of the state in Eq. (2). Here  $N_1' = [2(1 - e^{-4|\alpha|^2})]^{-\frac{1}{2}}$ .

In next step, we let the photons in spatial modes  $1_2$  and  $2_4$  pass through  $BS_7$ , and let the photons in spatial modes  $5_4$  and  $6_2$  pass through  $BS_8$ , respectively. The state  $|\psi\rangle_{2_1}$  combined with the state in spatial modes  $1_2$  and  $6_2$  becomes

$$\begin{aligned} |\psi\rangle_{2_2} = & |\psi\rangle_{2_1} \otimes \frac{1}{\sqrt{2}} N_0 (|\alpha\rangle_{1_2} + |-\alpha\rangle_{1_2}) \otimes \frac{1}{\sqrt{2}} N_0 (|\alpha\rangle_{6_2} + |-\alpha\rangle_{6_2}) \\ \rightarrow & \frac{1}{2\sqrt{2}} N_0^2 N_1'^2 [ (|\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_5}|\alpha\rangle_{3_7} + |0\rangle_{1_3}|-\sqrt{2}\alpha\rangle_{2_5}|\alpha\rangle_{3_7} + |0\rangle_{1_3}|\sqrt{2}\alpha\rangle_{2_5}|-\alpha\rangle_{3_7} \\ & + |-\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_5}|-\alpha\rangle_{3_7}) (|\alpha\rangle_{4_7}|\sqrt{2}\alpha\rangle_{5_5}|0\rangle_{6_3} + |-\alpha\rangle_{4_7}|0\rangle_{5_5}|-\sqrt{2}\alpha\rangle_{6_3} \\ & + |\alpha\rangle_{4_7}|0\rangle_{5_5}|\sqrt{2}\alpha\rangle_{6_3} + |-\alpha\rangle_{4_7}|-\sqrt{2}\alpha\rangle_{5_5}|0\rangle_{6_3}) ] \\ & + \frac{1}{2\sqrt{2}} N_0^2 N_1'^2 [ (|\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_5}|\alpha\rangle_{3_7} + |0\rangle_{1_3}|-\sqrt{2}\alpha\rangle_{2_5}|\alpha\rangle_{3_7} - |0\rangle_{1_3}|\sqrt{2}\alpha\rangle_{2_5}|-\alpha\rangle_{3_7} \\ & - |-\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_5}|-\alpha\rangle_{3_7}) (|\alpha\rangle_{4_7}|\sqrt{2}\alpha\rangle_{5_5}|0\rangle_{6_3} - |-\alpha\rangle_{4_7}|0\rangle_{5_5}|-\sqrt{2}\alpha\rangle_{6_3} \\ & + |\alpha\rangle_{4_7}|0\rangle_{5_5}|\sqrt{2}\alpha\rangle_{6_3} - |-\alpha\rangle_{4_7}|-\sqrt{2}\alpha\rangle_{5_5}|0\rangle_{6_3}) ]. \end{aligned} \quad (15)$$

Finally, we choose the cases that both spatial modes  $2_5$  and  $5_5$  have no photon, and let the state photons in spatial mode  $1_3$  pass through  $BS_9$ , and the photons in spatial mode  $6_3$  pass through  $BS_{10}$ , respectively. The state  $|\psi\rangle_{2_2}$  will ultimately collapse to state

$$\begin{aligned} |\psi\rangle_{2_3} \rightarrow & \frac{1}{2\sqrt{2}} N_0^2 N_1'^2 [ (|\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_5}|\alpha\rangle_{3_7} + |-\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_5}|-\alpha\rangle_{3_7}) \\ & \otimes (|-\alpha\rangle_{4_7}|0\rangle_{5_5}|-\sqrt{2}\alpha\rangle_{6_3} + |\alpha\rangle_{4_7}|0\rangle_{5_5}|\sqrt{2}\alpha\rangle_{6_3}) \\ & + \frac{1}{2\sqrt{2}} N_0^2 N_1'^2 [ (|\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_5}|\alpha\rangle_{3_7} - |-\sqrt{2}\alpha\rangle_{1_3}|0\rangle_{2_5}|-\alpha\rangle_{3_7}) ] \\ & \otimes (|\alpha\rangle_{4_7}|0\rangle_{5_5}|\sqrt{2}\alpha\rangle_{6_3} - |-\alpha\rangle_{4_7}|0\rangle_{5_5}|-\sqrt{2}\alpha\rangle_{6_3}) ] \\ \rightarrow & \frac{1}{2\sqrt{2}} N_0^2 N_1'^2 [ (|\alpha\rangle_{1_4}|\alpha\rangle_{2_6}|\alpha\rangle_{3_7} + |-\alpha\rangle_{1_4}|-\alpha\rangle_{2_6}|-\alpha\rangle_{3_7}) \\ & \otimes (|\alpha\rangle_{4_7}|\alpha\rangle_{5_5}|\alpha\rangle_{6_4} + |-\alpha\rangle_{4_7}|-\alpha\rangle_{5_5}|-\alpha\rangle_{6_4}) ] \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{2\sqrt{2}} N_0^2 N_1^2 [(|\alpha\rangle_{14} |\alpha\rangle_{26} |\alpha\rangle_{37} - |-\alpha\rangle_{14} |-\alpha\rangle_{26} |-\alpha\rangle_{37}) \\
& \otimes (|\alpha\rangle_{47} |\alpha\rangle_{55} |\alpha\rangle_{64} - |-\alpha\rangle_{47} |-\alpha\rangle_{55} |-\alpha\rangle_{64})].
\end{aligned} \tag{16}$$

State in Eq. (16) is the obtained state as shown in Eq. (13) if  $e^{-2|\alpha|^2} \rightarrow 0$ .

### III. GENERATION OF C-GHZ-TYPE ECS WITH ARBITRARY $N$ AND $m$

It is straightforward to extend this protocol to the case of generation of the C-GHZ-type ECS with arbitrary  $N$  and  $m$ . Generally, the whole protocol can be divided into two steps. In the first step, we generate a GHZ-type ECS

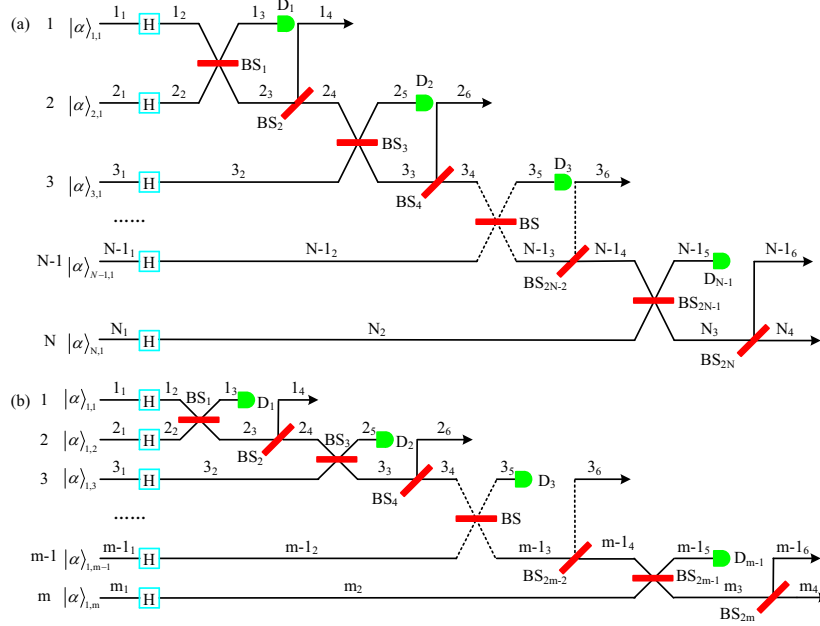


FIG. 3: Schematic diagram of generating the C-GHZ-type ECS with arbitrary  $N$  and  $m$ . (a) Schematic diagram shows the generation of GHZ-type ECS. (b) Schematic diagram shows the generation of logical bits for the first coherent state in the GHZ-type ECS.

with  $N$  coherent states. As shown in Fig. 3 (a), we prepare  $N$  coherent states in the spatial modes  $1_1, 2_1, \dots, N_1$ . Similar to the previous section, we first generate Bell-type ECS in the spatial modes  $1_4$  and  $2_4$  of the form

$$|\Psi\rangle_{11} = N_1(|\alpha\rangle_{14} |\alpha\rangle_{24} + |-\alpha\rangle_{14} |-\alpha\rangle_{24}), \tag{17}$$

under the condition that single-photon detector  $D_1$  detect no photon. Then, we combine the state  $|\Psi\rangle_{11}$  with the coherent state in spatial mode  $3_2$ . The whole system can be written as

$$\begin{aligned}
|\Psi\rangle_{12} &= |\Psi\rangle_{11} \otimes \frac{1}{\sqrt{2}} N_0 (|\alpha\rangle_{32} + |-\alpha\rangle_{32}) \\
&= \frac{1}{\sqrt{2}} N_0 N_1 (|\alpha\rangle_{14} |\alpha\rangle_{24} |\alpha\rangle_{32} + |-\alpha\rangle_{14} |-\alpha\rangle_{24} |\alpha\rangle_{32} \\
&\quad + |\alpha\rangle_{14} |\alpha\rangle_{24} |-\alpha\rangle_{32} + |-\alpha\rangle_{14} |-\alpha\rangle_{24} |-\alpha\rangle_{32}).
\end{aligned} \tag{18}$$

We let the photons in spatial modes  $2_4$  and  $3_2$  pass through  $BS_3$  and choosing the case that the spatial mode  $2_5$  has no photon. The state can be written as

$$|\Psi\rangle_{13} = \frac{1}{\sqrt{2}} N_0 N_1 (|\alpha\rangle_{14} |\sqrt{2}\alpha\rangle_{25} |0\rangle_{33} + |-\alpha\rangle_{14} |0\rangle_{25} |-\sqrt{2}\alpha\rangle_{33})$$

$$\begin{aligned}
& + |\alpha\rangle_{14}|0\rangle_{25}|\sqrt{2}\alpha\rangle_{33} + |-\alpha\rangle_{14}|-\sqrt{2}\alpha\rangle_{25,1}|0\rangle_{33}) \\
& \rightarrow \frac{1}{2}N_0N_1(|-\alpha\rangle_{14}|0\rangle_{25}|\sqrt{2}\alpha\rangle_{33} + |\alpha\rangle_{14}|0\rangle_{25}|\sqrt{2}\alpha\rangle_{33}).
\end{aligned} \tag{19}$$

Let the photons in spatial mode  $3_3$  pass through  $BS_4$ , and then the state will be described as

$$|\Psi\rangle_{13} = N_2(|\alpha\rangle_{14}|\alpha\rangle_{26}|\alpha\rangle_{34} + |-\alpha\rangle_{14}|-\alpha\rangle_{26}|-\alpha\rangle_{34}). \tag{20}$$

Here  $N_2 = [2(1 \pm e^{-6|\alpha|^2})]^{-\frac{1}{2}}$ . Following the same principle, by add another coherent states in spatial modes  $3_1, 4_1, \dots, N_1$ , we can get the GHZ-type ECS as

$$\begin{aligned}
|\Psi\rangle &= N_N(|\alpha\rangle_{14}|\alpha\rangle_{26}|\alpha\rangle_{36} \cdots |\alpha\rangle_{N-26}|\alpha\rangle_{N-16}|\alpha\rangle_{N4} \\
&+ |-\alpha\rangle_{14}|-\alpha\rangle_{26}|-\alpha\rangle_{36} \cdots |-\alpha\rangle_{N-26}|-\alpha\rangle_{N-16}|-\alpha\rangle_{N4}) \\
&= N_N(|\alpha\rangle^{\otimes N} + |-\alpha\rangle^{\otimes N}).
\end{aligned} \tag{21}$$

Here  $N_N = [2(1 \pm e^{-2N|\alpha|^2})]^{-\frac{1}{2}}$ . In the second step, we will generate the GHZ-type ECS on each qubit. We rewrite the state  $|\Psi\rangle$  as

$$|\Psi\rangle_1 = N_N(|\alpha\rangle_{1,1}|\alpha\rangle^{\otimes N-1} + |-\alpha\rangle_{1,1}|-\alpha\rangle^{\otimes N-1}). \tag{22}$$

Here the first "1" in subscript "1, 1" means the first logic qubit. The second "1" means the first physical qubit. As shown in Fig. 3 (b), we first perform the Hadamard operation on the qubit 1, 1 and make the state  $|\Psi\rangle_1$  becomes

$$\begin{aligned}
|\Psi\rangle_1 &\rightarrow |\Psi\rangle_2 = \frac{1}{\sqrt{2}}N_NN_0(|\alpha\rangle_{1,12} + |-\alpha\rangle_{1,12})|\alpha\rangle^{\otimes N-1} \\
&+ \frac{1}{\sqrt{2}}N_NN'_0(|\alpha\rangle_{1,12} - |-\alpha\rangle_{1,12})|-\alpha\rangle^{\otimes N-1}.
\end{aligned} \tag{23}$$

The state in Eq.(23) combined with the single coherent state in the spatial mode  $2_2$  can be written as

$$|\Phi\rangle_1 = |\Psi\rangle_2 \otimes \frac{1}{\sqrt{2}}N_0(|\alpha\rangle_{1,22} + |-\alpha\rangle_{1,22}). \tag{24}$$

Then, we let the photons in spatial modes  $1_2$  and  $2_2$  pass through  $BS_1$ . The state will be

$$\begin{aligned}
|\Phi\rangle_2 &= \frac{1}{2}N_0^2N_N[(|\sqrt{2}\alpha\rangle_{1,13}|0\rangle_{1,23} + |0\rangle_{1,13}|\sqrt{2}\alpha\rangle_{1,23} \\
&+ |0\rangle_{1,13}|-\sqrt{2}\alpha\rangle_{1,23} + |-\sqrt{2}\alpha\rangle_{1,13}|0\rangle_{1,23})|\alpha\rangle^{\otimes N-1}] \\
&+ \frac{1}{2}N_NN_0N'_0[(|\sqrt{2}\alpha\rangle_{1,13}|0\rangle_{1,23} + |0\rangle_{1,13}|\sqrt{2}\alpha\rangle_{1,23} \\
&- |0\rangle_{1,13}|-\sqrt{2}\alpha\rangle_{1,23} - |-\sqrt{2}\alpha\rangle_{1,13}|0\rangle_{1,23})|-\alpha\rangle^{\otimes N-1}].
\end{aligned} \tag{25}$$

By choosing the cases where the spatial mode  $1_3$  has no photon and making the photons in spatial mode  $2_2$  pass through  $BS_2$ , we will obtain the state

$$\begin{aligned}
|\Phi\rangle_3 &= \frac{1}{2}N_0^2N_N(|\alpha\rangle_{1,14}|\alpha\rangle_{1,24} + |-\alpha\rangle_{1,14}|-\alpha\rangle_{1,24})|\alpha\rangle^{\otimes N-1} \\
&+ \frac{1}{2}N_NN_0N'_0(|\alpha\rangle_{1,14}|\alpha\rangle_{1,24} - |-\alpha\rangle_{1,14}|-\alpha\rangle_{1,24})|-\alpha\rangle^{\otimes N-1}.
\end{aligned} \tag{26}$$

Following the same principle, by adding another coherent states in spatial modes  $3_1, 4_1, \dots, m_1$ , we can obtain the state

$$\begin{aligned}
|\Phi\rangle_{1,m} &= \frac{1}{2}N_0^{m-1}N_N(|\alpha\rangle_{1,14}|\alpha\rangle_{1,26}|\alpha\rangle_{1,36} \cdots |\alpha\rangle_{1,m-16}|\alpha\rangle_{1,m4} \\
&+ |-\alpha\rangle_{1,14}|-\alpha\rangle_{1,26}|-\alpha\rangle_{1,36} \cdots |-\alpha\rangle_{1,m-16}|-\alpha\rangle_{1,m4})|\alpha\rangle^{\otimes N-1} \\
&+ \frac{1}{2}N_0N_NN_0'^{m-2}(|\alpha\rangle_{1,14}|\alpha\rangle_{1,26}|\alpha\rangle_{1,36} \cdots |\alpha\rangle_{1,m-16}|\alpha\rangle_{1,m4} \\
&- |-\alpha\rangle_{1,14}|-\alpha\rangle_{1,26}|-\alpha\rangle_{1,36} \cdots |-\alpha\rangle_{1,m-16}|-\alpha\rangle_{1,m4})|-\alpha\rangle^{\otimes N-1} \\
&= \frac{1}{2}N_0^{m-1}N_N(|\alpha\rangle_1^{\otimes m} + |-\alpha\rangle_1^{\otimes m})|\alpha\rangle^{\otimes N-1} \\
&+ \frac{1}{2}N_0N_NN_0'^{m-2}(|\alpha\rangle_1^{\otimes m} - |-\alpha\rangle_1^{\otimes m})|-\alpha\rangle^{\otimes N-1}.
\end{aligned} \tag{27}$$

In the same way, by adding  $m - 1$  coherent state in each logic qubit, we can finally obtain the C-GHZ-type ECS

$$\begin{aligned} |\Phi\rangle &= (N_0^{m-2})^{\otimes N} ( (|\alpha\rangle^{\otimes m} + |-\alpha\rangle^{\otimes m})^{\otimes N} + (N_0'^{m-2})^N (|\alpha\rangle^{\otimes m} - |-\alpha\rangle^{\otimes m})^{\otimes N} ) \\ &\approx \frac{1}{\sqrt{2}} (|GHZ_m^+\rangle^{\otimes N} + |GHZ_m^-\rangle^{\otimes N}). \end{aligned} \quad (28)$$

#### IV. CONCLUSION

We have designed the protocol to construct the arbitrary C-GHZ-type ECS from single coherent states. Generally speaking, the protocol can be divided into two steps. In the first step, we prepare the N-GHZ-type ECS. Second, we prepare arbitrary C-GHZ-type ECS from N-GHZ-type ECS. We should point out that this protocol cannot obtain the exact the C-GHZ-type ECS. The main reason is that the  $\langle -\alpha|\alpha\rangle = e^{-2|\alpha|^2} \neq 0$ . As shown in Eq. (12), we can obtain the state in Eq. (2) under the condition that  $e^{-2|\alpha|^2} \rightarrow 0$ , if  $\alpha$  is large enough.

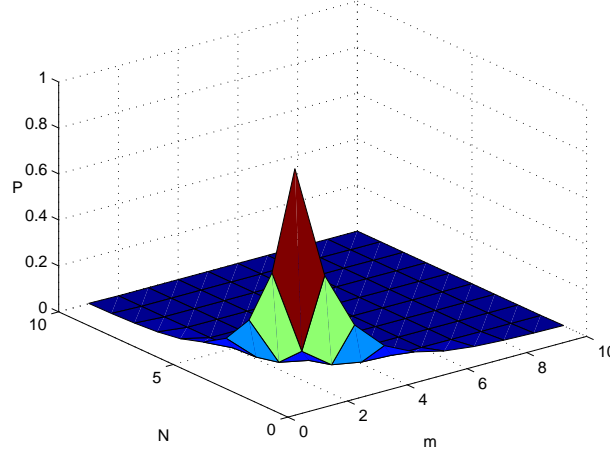


FIG. 4: Schematic diagram showing the success probability  $P$  of the protocol. The  $\alpha$  is the amplitude of the coherent state. Here, we let the  $\alpha=10$ .

The total success probability  $P$  of the protocol can be written as

$$P = \frac{1}{2^{Nm-1}}. \quad (29)$$

From Fig. 4,  $P$  decreases with both  $m$  and  $N$ . Here we choose  $\alpha = 10$ . In this protocol, the success case is the single-photon detector does not register any photon. Such selection condition provides us a good advantage to realize this protocol. In conventional quantum information protocols based on linear optics, the success condition is that the single-photon detector register one and only one photon. However, once the photon is detected, it is also destroyed and cannot be used in the future quantum information processing. Interestingly, by selecting the case that the single-photon detector does not register photon, the state can be remained. Certainly, we should point out that the coherent state  $|\alpha\rangle$  also has the probability to make the single-photon detector do not register the photon. It will induce the error. Fortunately, the error probability is too small and can be ignored, if  $\alpha$  is large. For example, when  $\alpha = 2$ , the error probability  $|\langle 0|\alpha\rangle|^2$  is  $1.1 \times 10^{-7}$ .

In summary, we have presented a protocol for preparing the C-GHZ-type ECS. We first describe the protocol in the case of  $N = m = 2$  and  $N = 2, m = 3$  respectively. Then we describe the protocol in the case of  $N = m = 3$  and finally extend the protocol to the arbitrary number of  $N$  and  $m$ . This protocol may be useful in the future of long-distance quantum communications based on C-GHZ state.

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